COURSE CONTENT FOR TRANSIT TRAINING

1.1 ANALYTICAL SOLAR CELL MODELING APPROACHES

1.1.1 Introduction

In order to reduce greenhouse gas emissions and mitigate climate changes, electrical power systems are actively incorporating renewable energy sources alongside conventional ones. Among these, solar energy is the most prevalent [1]. Photovoltaic (PV) devices, or solar cells, work on principle of directly converting solar radiation into electrical energy. Because of the efficiency and growing affordability, the PV technology is now the dominant and fastest-expanding form of renewable energy worldwide, accounting for about 60% of total renewable energy investments in 2022. Thus, can be clearly conducted that the photovoltaic technology has very important role in the transformation of global energy.

Taking into account that the application of solar energy as a renewable source contains numerous advantages, including reduced greenhouse gas emissions, less reliance on fossil fuels, and versatile applications, it also introduces several challenges. The most significant difficulties include its non-stationary nature, dependence on weather conditions, energy storage challenges, and the high initial costs associated with the use of relatively expensive inverter equipment [2]. Moreover, total production costs are increased due to the need for maximum power point tracking, which is required for enhancing the efficiency [3].

The current–voltage (*I*–*V*) characteristic curve is the best way to represent the electrical output of a solar cell (modules and panels), in terms of electrical energy production via photovoltaic modules [4]. Accurate mathematical representation of this curve is crucial and must consider all relevant factors, such as the inherent parameters of the cell and external influences, such as radiation (G) and temperature (T) [5]. Considering the nonlinearity of the *I*–*V* characteristic, it is obvious why various models are employed to represent the equivalent circuit of a solar cell, each differing in complexity and accuracy.

The *I*–*V* characteristics of solar cells are characterized by high nonlinearity [4,6]. Each model of solar cell is represented with set of parameters, which will be described in details in following section. However, determining the parameters of solar cells can be approached in two main ways:

- Classical methods, which may be either analytical [7] or numerical [8],
- Modern metaheuristic methods [9, 10].

Analytical methods, which were initially most common in the analysis of solar cell operation, offer precise descriptions of the physical behaviors of the models they study. They provide exact solutions for problems that can be mathematically defined by specific equations [7]. However, due to the complex nature of the equations describing solar cells′ *I*–*V* characteristics, various approximations must be introduced. These approximations, while necessary, can significantly diminish the accuracy of the solutions and neglect some of very important physical processes in the cell.

With advances in the computer technology, including the CPU speed and memory space of the computers, numerical methods have gained prominence. These methods have largely surpassed analytical methods in terms of accuracy, as they handle the complexity of solar cell models more effectively [8]. Their use is, however, bounded by the computational power available, the accuracy of results based on the iteration step, and the time required for computations.

Metaheuristic methods today offer the most promising results in determining the parameters of the equivalent circuits of solar cells. These methods are recognized for their efficiency in optimizing highly complex and nonlinear problems. Their principal advantage lies in their ability to explore a vast array of potential solutions extensively, thereby identifying optimal or near-optimal parameter sets for the models in question. In the recent scientific research, these methods are the most common way to determine the parameters of the equivalent circuits of solar cells. Only small part of this research are is presented in [9] and [10].

1.1.2 Classical models of solar cells

Ideal solar cell model, that does not include any kind of losses in the cell, consists of current source and the diode. However, real model of solar cell must include the losses that occur during the operational mode. Mostly accepted approach is to use two resistances to model the losses: serial resistance R_S-resistance of metal grids, contacts, and current-collecting wires, and parallel resistance R_P that exists because solar cells are made of large-area wafers and from large thin-film material. In the available literature three most common solar cell models are used. Depending on the number of diodes in the equivalent circuit, there are single-diode model (SDM), double-diode model (DDM), and triple-diode model (TDM). Equivalent circuit of mentioned models are depicted in Fig. 1.

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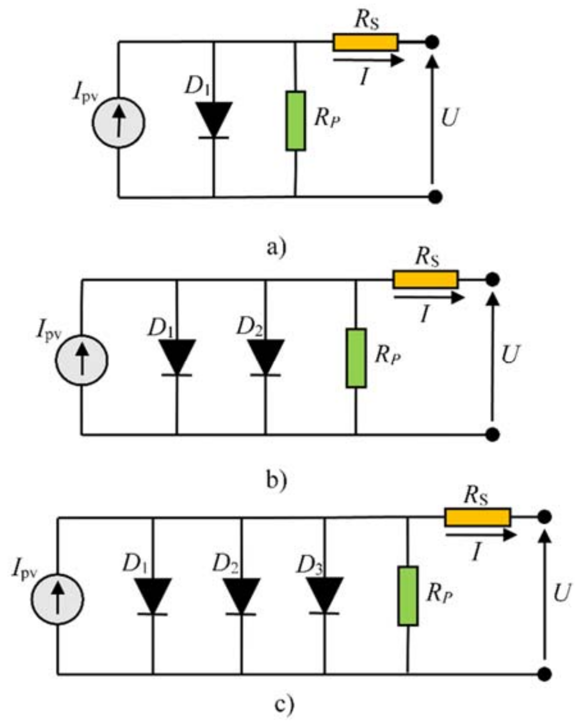

Fig. 1. Solar PV cell equivalent circuits: (a) SDM, (b) DDM, and (c) TDM.

The current-voltage *(I-V*) expression can be derived for all three equivalent circuits [9].

Observing the equivalent circuit of the SDM, current can be expressed as follows:

$$I = I_{PV} - \frac{V + IR_S}{R_P} - I_0\left(e^{\frac{V+IR_S}{nV_{th}}} - 1\right). \tag{1}$$

Current-voltage characteristic for the equivalent circuit of the DDM can be expressed with the following equation:

$$I = I_{PV} - \frac{V + IR_S}{R_P} - I_{01}\left(e^{\frac{V+IR_S}{n_1V_{th}}} - 1\right) - I_{02}\left(e^{\frac{V+IR_S}{n_2V_{th}}} - 1\right). \tag{2}$$

Finally, the *I-V* expression for the equivalent circuit of the TDM of PV cell is expressed as follows:

$$I = I_{PV} - \frac{V + IR_S}{R_P} - I_{01}\left(e^{\frac{V+IR_S}{n_1V_{th}}} - 1\right) - I_{02}\left(e^{\frac{V+IR_S}{n_2V_{th}}} - 1\right) - I_{03}\left(e^{\frac{V+IR_S}{n_3V_{th}}} - 1\right). \tag{3}$$

The terms from previous equations have the following meaning:

-I_{PV} denotes the photogenerated current,

-I_{0j} denotes the reverse saturation current of the *j*-th diode

-n_j represents the ideality factor of the *j*-th diode,

-V_{th} = $K_B T/q$ is the thermal voltage (K_B is the Boltzmann constant, *T* is the temperature in Kelvin and *q* is the charge of the electron),

-R_s represents the series resistance

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-R_p is parallel resistance.

1.1.3 Single-diode model (SDM) of solar cell

In this part, the most common, and most simple model, single-diode model, is analyzed. Firstly, the classical single-diode model of solar cell is presented, and afterwards modified and nonlinear variants of the single-diode model are presented.

1.1.3.1 Classical single-diode model of solar cell

As it can be seen, equations for all solar cell models are highly nonlinear, i.e., transcendental. The analytical solution of current in function of voltage for SDM is as follows [10]:

$$I = \frac{R_P\left(I_{pv} + I_0\right) - U}{R_S + R_P} - \frac{n_1 \cdot V_{th}}{R_S} W(\alpha_S), \tag{4}$$

where

$$\alpha_S = \frac{I_0 R_P R_S}{n_1 \cdot V_{th}\left(R_S + R_P\right)} \exp\left(\frac{R_P\left(R_S I_{pv} + R_S I_0 + U\right)}{n_1 \cdot V_{th}\left(R_S + R_P\right)}\right) \tag{5}$$

and where *W*() is Lambert W function.

1.1.3.2 Modified single-diode model of solar cell

As mentioned in the previous part, losses in the PV cell are typically modeled with two resistances- serial R_S and parallel R_P. This model introduces new resistance R_{SD} in order to represent the power losses due to the current flowing through p-n junction. The equivalent circuit of the proposed improved SDM is given in Fig. 2 [4].

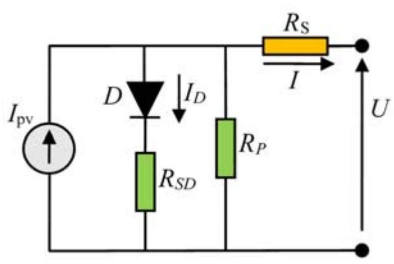

Fig. 2. Proposed - improved SDM

The equation for sum of currents in this model is as follows:

$$I_{pv} = I_D + \frac{U + IR_S}{R_P} + I, \tag{6}$$

where

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$$I_D = I_0\left(e^{\frac{V_D}{n_1 \times V_{th}}} - 1\right) \quad (7)$$

Also, the voltage equation for this circuit is as follows:

$$V_D + R_{SD} I_D = U + IR_S. \quad (8)$$

By combining the previous equations, the expression for current can be derived in the following form:

$$I = \frac{R_P}{R_S + R_P}\left(I_{pv} + I_{01} - \frac{U}{R_P} - x\right), \quad (9)$$

where x is the solution of the Lambert W function in the following form:

$$x = \beta \cdot \exp(-x). \quad (10)$$

In this equation,

$$\beta = I_2 \frac{b}{a} \cdot \exp\left(\frac{b}{a} \cdot I_1\right), \quad (11)$$

where

$$\begin{aligned} a &= 1 + \frac{R_S}{R_P} \\ b &= \frac{R_S}{nV_t}\left(1 + \frac{R_{SD}}{R_P} + \frac{R_{SD}}{R_S}\right) \\ I_1 &= I_{pv} + I_{01} - \frac{U}{R_P} \\ I_2 &= I_{01} \exp\left(\frac{1}{nV_t}\left(U - R_{SD} I_{pv} + \frac{R_{SD}}{R_P} U\right)\right) \end{aligned} \quad (12)$$

Based on previous researches [11-13], where it is clearly shown that the STFT have a significant advantage over the Taylor series, the analytical solution of current-voltage relation for improved solar cell model can be represented as follows:

$$I = \frac{R_P}{R_S + R_P}\left(I_{pv} + I_{01} - \frac{U}{R_P} - \beta \frac{\sum_{k=0}^{M} \frac{\beta^k (M-k)^k}{k!}}{\sum_{k=0}^{M+1} \frac{\beta^k (M+1-k)^k}{k!}}\right) \quad (13)$$

where M represents a positive integer.

The power-voltage dependences can be expressed as follows:

$$\begin{aligned} P &= U \cdot I \\ P &= \frac{R_P U}{R_S + R_P}\left(I_{pv} + I_{01} - \frac{U}{R_P} - \beta \frac{\sum_{k=0}^{M} \frac{\beta^k (M-k)^k}{k!}}{\sum_{k=0}^{M+1} \frac{\beta^k (M+1-k)^k}{k!}}\right) \end{aligned} \quad (14)$$

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The voltage corresponding to the maximum power delivered (U_{mp}) by the device can be determined as follows:

$$\left.\left(\frac{\partial P(U)}{\partial U}\right)\right|_{U=U_{mp}} = 0, \tag{15}$$

while from

$$P_{mp} = U_{mp} \cdot I_{mp}, \tag{16}$$

also, the current corresponding to the maximum power can be calculated, where P_{mp} is maximum power point of the solar cell.

1.1.3.3 *Nonlinear single-diode model of solar cell with linear change of resistance*

The modified SDM with voltage dependent series resistance is proposed in [14] to elucidate the electrical behavior of Organic Solar Cells. On this way, the standard SDM uses a voltage-dependent series resistance to enhance the modeling accuracy while benefiting from the simplicity of the equivalent circuit. The reasons for the introduction of the voltage dependence of the series resistance, the physical interpretation and others are described in [14]. Basically, the application of this modification is mainly related to internal processes of charge extraction and charge transport. In [14] it is concluded that a voltage dependent series resistance provides a good knowledge about the behavior of the organic solar cell at different applied voltage regions.

Based on this paper, in this section, three novel SDM circuit are presented (see Fig. 3) [15]. These circuits, in contrast with standard SDM, have voltage dependent series resistance (Fig. 3a), voltage dependent parallel resistance (Fig. 3b) and voltage dependent of both resistances (Fig. 3c).

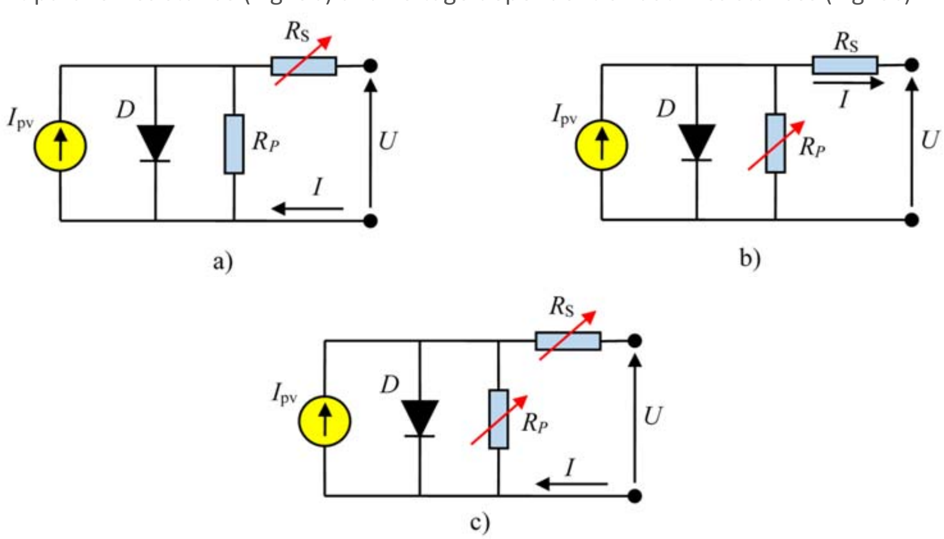

Fig. 3. Proposed Nonlinear Single diode models of the solar cell a) SDM_{RP}, SDM_{RS}, SDM_{RPRS}

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For Fig. 3a, the analytical solution for current in function of voltage is as follows:

$$I_{n-R_s} = \frac{R_P\left(I_{pv}+I_0\right)-U}{R_{S0}\left(1+k_{n-R_s}\cdot U\right)+R_P} - \frac{nV_t}{R_{S0}\left(1+k_{n-R_s}\cdot U\right)}\cdot W(\beta_{n-R_s}), \tag{17}$$

where

$$\beta_{n-R_s} = A_{n-R_s}\exp\left(\frac{R_P\left(R_{S0}\left(1+k_{n-R_s}\cdot U\right)\cdot I_{pv}+R_{S0}\left(1+k_{n-R_s}\cdot U\right)\cdot I_0+U\right)}{nV_t\left(R_{S0}\left(1+k_{n-R_s}\cdot U\right)+R_P\right)}\right), \tag{18}$$

$$A_{n-R_s} = \frac{I_0 R_P R_{S0}\left(1+k_{n-R_s}\cdot U\right)}{nV_t\left(R_{S0}\left(1+k_{n-R_s}\cdot U\right)+R_P\right)}. \tag{19}$$

Applying the STFT [11-13], the current voltage expression is as follows:

$$I = \frac{R_P\left(I_{pv}+I_0\right)-U}{R_{S0}\left(1+k_{n-R_s}\cdot U\right)+R_P} - \frac{I_0 R_P}{R_{S0}\left(1+k_{n-R_s}\cdot U\right)+R_P}\mathrm{e}^{\frac{R_P\left(R_{S0}\left(1+k_{n-R_s}\cdot U\right)\cdot I_{pv}+R_{S0}\left(1+k_{n-R_s}\cdot U\right)\cdot I_0+U\right)}{nV_t\left(R_{S0}\left(1+k_{n-R_s}\cdot U\right)+R_P\right)}}\cdot$$
$$\cdot\left(\frac{\sum_{k=0}^{M}\frac{\left(\frac{I_0 R_P R_{S0}\left(1+k_{n-R_s}\cdot U\right)}{nV_t\left(R_{S0}\left(1+k_{n-R_s}\cdot U\right)+R_P\right)}\mathrm{e}^{\frac{R_P\left(R_{S0}\left(1+k_{n-R_s}\cdot U\right)\cdot I_{pv}+R_{S0}\left(1+k_{n-R_s}\cdot U\right)\cdot I_0+U\right)}{nV_t\left(R_{S0}\left(1+k_{n-R_s}\cdot U\right)+R_P\right)}}\right)^k\left(M-k\right)^k}{k!}}{\sum_{k=0}^{M+1}\frac{\left(\frac{I_0 R_P R_{S0}\left(1+k_{n-R_s}\cdot U\right)}{nV_t\left(R_{S0}\left(1+k_{n-R_s}\cdot U\right)+R_P\right)}\mathrm{e}^{\frac{R_P\left(R_{S0}\left(1+k_{n-R_s}\cdot U\right)\cdot I_{pv}+R_{S0}\left(1+k_{n-R_s}\cdot U\right)\cdot I_0+U\right)}{nV_t\left(R_{S0}\left(1+k_{n-R_s}\cdot U\right)+R_P\right)}}\right)^k\left(M+1-k\right)^k}{k!}}\right) \tag{20}$$

For Fig. 3b, the analytical solution for current in function of voltage is as follows:

$$I_{n-R_P} = \frac{R_{P0}\left(1+K_{n-R_P}\cdot U\right)\left(I_{pv}+I_0\right)-U}{R_S+R_{P0}\left(1+K_{n-R_P}\cdot U\right)} - \frac{nV_t}{R_S}\cdot W(\beta_{n-R_P}), \tag{21}$$

where

$$\beta_{n-R_P} = A_{n-R_P}\exp\left(\frac{R_{P0}\left(1+K_{n-R_P}\cdot U\right)\left(R_S I_{pv}+R_S I_0+U\right)}{nV_t\left(R_S+R_{P0}\left(1+K_{n-R_P}\cdot U\right)\right)}\right),$$
$$A_{n-R_P} = \frac{I_0 R_S R_{P0}\left(1+K_{n-R_P}\cdot U\right)}{nV_t\left(R_S+R_{P0}\left(1+K_{n-R_P}\cdot U\right)\right)}. \tag{22}$$

Applying the STFT, the current voltage expression is as follows:

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$$I_{n-R_P}=\frac{R_{P0}\left(1+K_{n-R_P}\cdot U\right)\left(I_{pv}+I_0\right)-U}{R_S+R_{P0}\left(1+K_{n-R_P}\cdot U\right)}-\frac{I_0R_{P0}\left(1+K_{n-R_P}\cdot U\right)}{R_S+R_{P0}\left(1+K_{n-R_P}\cdot U\right)}\mathrm{e}^{\frac{R_{P0}\left(1+K_{n-R_P}\cdot U\right)\left(R_SI_{pv}+R_SI_0+U\right)}{nV_t\left(R_S+R_{P0}\left(1+K_{n-R_P}\cdot U\right)\right)}}\cdot\left(\frac{\sum_{k=0}^{M}\frac{\left(\frac{I_0R_SR_{P0}\left(1+K_{n-R_P}\cdot U\right)}{nV_t\left(R_S+R_{P0}\left(1+K_{n-R_P}\cdot U\right)\right)}\mathrm{e}^{\frac{R_{P0}\left(1+K_{n-R_P}\cdot U\right)\left(R_SI_{pv}+R_SI_0+U\right)}{nV_t\left(R_S+R_{P0}\left(1+K_{n-R_P}\cdot U\right)\right)}}\right)^k\left(M-k\right)^k}{k!}}{\sum_{k=0}^{M+1}\frac{\left(\frac{I_0R_SR_{P0}\left(1+K_{n-R_P}\cdot U\right)}{nV_t\left(R_S+R_{P0}\left(1+K_{n-R_P}\cdot U\right)\right)}\mathrm{e}^{\frac{R_{P0}\left(1+K_{n-R_P}\cdot U\right)\left(R_SI_{pv}+R_SI_0+U\right)}{nV_t\left(R_S+R_{P0}\left(1+K_{n-R_P}\cdot U\right)\right)}}\right)^k\left(M+1-k\right)^k}{k!}}\right) \quad (23)$$

Finally, for Fig. 3c, the analytical solution for current in function of voltage is as follows:

$$I_{n-R_PR_S}=\frac{R_{P0}\left(1+K_{n-R_P}\cdot U\right)\left(I_{pv}+I_0\right)-U}{R_{S0}\left(1+k_{n-R_s}\cdot U\right)+R_{P0}\left(1+K_{n-R_P}\cdot U\right)}-\frac{nV_t}{R_{S0}\left(1+k_{n-R_s}\cdot U\right)}\cdot W(\beta_{n-R_PR_S}), \quad (24)$$

where

$$\beta_{n-R_PR_S}=A_{n-R_PR_S}\exp\left(\frac{R_{P0}\left(1+K_{n-R_P}\cdot U\right)\left(R_{S0}\left(1+k_{n-R_s}\cdot U\right)I_{pv}+R_{S0}\left(1+k_{n-R_s}\cdot U\right)\cdot I_0+U\right)}{nV_t\left(R_{S0}\left(1+k_{n-R_s}\cdot U\right)+R_{P0}\left(1+K_{n-R_P}\cdot U\right)\right)}\right),$$
$$A_{n-R_PR_S}=\frac{I_0\cdot R_{S0}\cdot R_{P0}\cdot\left(1+K_{n-R_P}\cdot U\right)\left(1+k_{n-R_s}\cdot U\right)}{nV_t\left(R_{S0}\left(1+k_{n-R_s}\cdot U\right)+R_{P0}\left(1+K_{n-R_P}\cdot U\right)\right)}. \quad (25)$$

Applying the STFT, the current voltage expression is as follows:

$$I_{n-R_PR_S}=\frac{R_{P0}\left(1+K_{n-R_P}\cdot U\right)\left(I_{pv}+I_0\right)-U}{R_{S0}\left(1+k_{n-R_s}\cdot U\right)+R_{P0}\left(1+K_{n-R_P}\cdot U\right)}-\frac{I_0\cdot R_{P0}\cdot\left(1+K_{n-R_P}\cdot U\right)}{\left(R_{S0}\left(1+k_{n-R_s}\cdot U\right)+R_{P0}\left(1+K_{n-R_P}\cdot U\right)\right)}\cdot\mathrm{e}^{\frac{R_{P0}\left(1+K_{n-R_P}\cdot U\right)\left(R_{S0}\left(1+k_{n-R_s}\cdot U\right)I_{pv}+R_{S0}\left(1+k_{n-R_s}\cdot U\right)\cdot I_0+U\right)}{nV_t\left(R_{S0}\left(1+k_{n-R_s}\cdot U\right)+R_{P0}\left(1+K_{n-R_P}\cdot U\right)\right)}}\cdot\left(\frac{\sum_{k=0}^{M}\frac{\left(\frac{I_0\cdot R_{S0}\cdot R_{P0}\cdot\left(1+K_{n-R_P}\cdot U\right)\left(1+k_{n-R_s}\cdot U\right)}{nV_t\left(R_{S0}\left(1+k_{n-R_s}\cdot U\right)+R_{P0}\left(1+K_{n-R_P}\cdot U\right)\right)}\mathrm{e}^{\frac{R_{P0}\left(1+K_{n-R_P}\cdot U\right)\left(R_{S0}\left(1+k_{n-R_s}\cdot U\right)I_{pv}+R_{S0}\left(1+k_{n-R_s}\cdot U\right)\cdot I_0+U\right)}{nV_t\left(R_{S0}\left(1+k_{n-R_s}\cdot U\right)+R_{P0}\left(1+K_{n-R_P}\cdot U\right)\right)}}\right)^k\left(M-k\right)^k}{k!}}{\sum_{k=0}^{M+1}\frac{\left(\frac{I_0\cdot R_{S0}\cdot R_{P0}\cdot\left(1+K_{n-R_P}\cdot U\right)\left(1+k_{n-R_s}\cdot U\right)}{nV_t\left(R_{S0}\left(1+k_{n-R_s}\cdot U\right)+R_{P0}\left(1+K_{n-R_P}\cdot U\right)\right)}\mathrm{e}^{\frac{R_{P0}\left(1+K_{n-R_P}\cdot U\right)\left(R_{S0}\left(1+k_{n-R_s}\cdot U\right)I_{pv}+R_{S0}\left(1+k_{n-R_s}\cdot U\right)\cdot I_0+U\right)}{nV_t\left(R_{S0}\left(1+k_{n-R_s}\cdot U\right)+R_{P0}\left(1+K_{n-R_P}\cdot U\right)\right)}}\right)^k\left(M+1-k\right)^k}{k!}}\right)$$

(26)

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1.1.4 Double-diode model (DDM) and triple-diode (TDM) models of solar cells

The analytical solution of current for DDM [16] is given as follows:

$$I = \frac{\left(I_{pv} + I_{01} + I_{02} - \frac{U}{R_P} - \frac{\Psi\left(1+\frac{R_s}{R_p}\right)}{\frac{R_s}{n_1 V_{th}}} \right)}{1+\frac{R_s}{R_p}}, \tag{27}$$

where Ψ is the solution of the following nonlinear equation:

$$\alpha_D + \beta_D \cdot \exp(\delta_D \cdot \Psi) = \Psi \cdot \exp(\Psi), \tag{28}$$

obtained using the following iterative equation:

$$^{(p)}\Psi = \mathrm{W}\left(\alpha_D + \beta_D \exp\left(\delta_D \cdot {}^{(p-1)}\Psi\right)\right), \tag{29}$$

where p stands for the current iteration, and W() denotes Lambert-W function.

In the previous equation, the terms are defined as follows:

$$\alpha_D = \frac{\frac{R_S}{n_1 \cdot V_{th}}}{\left(1+\frac{R_S}{R_P}\right)} \cdot I_{o1} \cdot \exp\left(\frac{U}{n_1 \cdot V_{th}}\right) \cdot \exp\left(\frac{\frac{R_S}{n_1 \cdot V_{th}}\left(I_{pv} + I_{01} + I_{02} - \frac{U}{R_P}\right)}{\left(1+\frac{R_S}{R_P}\right)} \right), \tag{30}$$

$$\beta_D = \frac{\frac{R_S}{n_1 \cdot V_{th}}}{\left(1+\frac{R_S}{R_P}\right)} \cdot I_{02} \cdot \exp\left(\frac{U}{n_2 \cdot V_{th}}\right) \cdot \exp\left(\frac{R_s}{n_2 \cdot V_{th}} \frac{\left(I_{pv} + I_{01} + I_{02} - \frac{U}{R_P}\right)}{\left(1+\frac{R_S}{R_P}\right)} \right), \tag{31}$$

$$\delta_D = 1 - \frac{n_1}{n_2}. \tag{32}$$

On the other side, the analytical solution of current for TDM [16] is as follows:

$$I = \frac{\left(I_{pv} + I_{01} + I_{02} + I_{03} - \frac{U}{R_P} - \frac{\mathrm{Z}\left(1+\frac{R_S}{R_P}\right)}{\frac{R_S}{n_1 \cdot V_t}} \right)}{1+\frac{R_S}{R_P}}, \tag{33}$$

where Z is the solution of the nonlinear equation:

$$\alpha_T + \beta_T \cdot \exp(\delta_T \cdot \mathrm{Z}) + \gamma_T \cdot \exp(\sigma_T \cdot \mathrm{Z}) = \mathrm{Z} \cdot \exp(\mathrm{Z}), \tag{34}$$

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obtained using the following iterative equation (where *p* stands for the current iteration, and W() denotes Lambert-W function):

$$ {}^{(p)}\mathrm{Z} = \mathrm{W}\left(\alpha_T + \beta_T \exp\left(\delta_T \cdot {}^{(p)}\mathrm{Z}\right) + \mathrm{Z}\exp\left(\sigma_T \cdot {}^{(p)}\mathrm{Z}\right)\right). \tag{35} $$

Different terms from the previous equation are defined as follows:

$$ \alpha_T = \frac{\frac{R_S}{n_1 \cdot V_{th}}}{\left(1+\frac{R_S}{R_P}\right)} \cdot I_{01} \cdot \exp\left(\frac{U}{n_1 \cdot V_{th}}\right) \cdot \exp\left(\frac{\frac{R_S}{n_1 \cdot V_{th}}\left(I_{pv} + I_{01} + I_{02} + I_{03} - \frac{U}{R_P}\right)}{\left(1+\frac{R_S}{R_p}\right)}\right), \tag{36} $$

$$ \beta_T = \frac{\frac{R_S}{n_1 \cdot V_{th}}}{\left(1+\frac{R_S}{R_P}\right)} \cdot I_{02} \cdot \exp\left(\frac{U}{n_2 \cdot V_t}\right) \cdot \exp\left(\frac{R_S}{n_2 \cdot V_{th}} \frac{\left(I_{pv} + I_{01} + I_{02} + I_{03} - \frac{U}{R_P}\right)}{\left(1+\frac{R_S}{R_P}\right)}\right), \tag{37} $$

$$ \gamma_T = \frac{\frac{R_S}{n_1 \cdot V_{th}}}{\left(1+\frac{R_S}{R_P}\right)} \cdot I_{03} \cdot \exp\left(\frac{U}{n_3 \cdot V_{th}}\right) \cdot \exp\left(\frac{R_S}{n_3 \cdot V_{th}} \frac{\left(I_{pv} + I_{01} + I_{02} + I_{03} - \frac{U}{R_P}\right)}{\left(1+\frac{R_s}{R_P}\right)}\right), \tag{38} $$

$$ \delta_T = 1 - \frac{n_1}{n_2}, \sigma_T = 1 - \frac{n_1}{n_3}. \tag{39} $$

The mentioned nonlinear equations do not have analytical solutions. However, in [16] original iterative procedures for solving the mentioned nonlinear equatios were proposed and tested.

The oldest research on the analytical modeling of *I-V* DDMs was proposed by Ortiz et al. [17] in 2012. On the basis of this research, the analytical *I-V* expression has the following form:

$$ \begin{aligned} I = {} & \frac{n_{1a}V_{th}}{a_1 R_S} W\left(\frac{a_1 R_S I_{01}}{n_{1a}V_{th}\left(1+\frac{R_S}{R_P}\right)} e^{\frac{V + R_S I_{PV} + a_1 R_S I_{01}}{n_{1a}V_{th}\left(1+\frac{R_S}{R_P}\right)}}\right) - \frac{I_{01}}{\left(1+\frac{R_S}{R_P}\right)} + \\ & \frac{n_{2a}V_{th}}{a_2 R_S} W\left(\frac{a_2 R_S I_{02}}{n_{2a}V_{th}\left(1+\frac{R_S}{R_P}\right)} e^{\frac{V + R_S I_{PV} + a_2 R_S I_{02}}{n_{2a}V_{th}\left(1+\frac{R_S}{R_P}\right)}}\right) - \frac{I_{02}}{\left(1+\frac{R_S}{R_P}\right)} + \frac{\frac{V}{R_P} - I_{PV}}{\left(1+\frac{R_S}{R_P}\right)}, \end{aligned} \tag{40} $$

where the following assumptions are made:

$$ \begin{aligned} & n_{1a} \approx n_1 \\ & n_{2a} \approx n_2 \\ & a_1 \approx a_2 \approx 1. \end{aligned} \tag{41} $$

In addition to Ortiz et al., there are a few other researchers in this field. In 2015, Lun et al. [18] proposed the following analytical *I-V* expression for solar cells, where W() denotes already mentioned Lambert-W function:

$$I = \frac{R_P\left(I_{PV} + I_{01} + I_{02}\right) - V}{R_S + R_P} - \frac{n_1 V_{th}}{2R_S} W\left(\xi_{V1}\right) - \frac{n_2 V_{th}}{2R_S} W\left(\xi_{V2}\right), \tag{42}$$

and individual terms are defined as follows:

$$\begin{aligned} \xi_{V1} &= \frac{R_S R_P\left(I_{01} + I_{02}\right)}{n_1 V_{th}\left(R_S + R_P\right)} e^{\frac{R_P\left(R_S I_{PV} + R_S I_{01} + R_S I_{02} + V\right)}{n_1 V_{th}\left(R_S + R_P\right)}} \\ \xi_{V2} &= \frac{R_S R_P\left(I_{01} + I_{02}\right)}{n_2 V_{th}\left(R_S + R_P\right)} e^{\frac{R_P\left(R_S I_{PV} + R_S I_{01} + R_S I_{02} + V\right)}{n_2 V_{th}\left(R_S + R_P\right)}}. \end{aligned} \tag{43}$$

In the mentioned paper, the relation between the solar cell voltage and current *V*=*V*(*I*) is also derived in terms of the Lambert W function as follows:

$$V = R_P\left(I_{PV} + I_{01} + I_{02} - I\right) - IR_S - \frac{n_1 V_{th}}{2} W\left(\xi_{I1}\right) - \frac{n_2 V_{th}}{2} W\left(\xi_{I2}\right), \tag{44}$$

where ξ_{I1} and ξ_{I2} are defined as follows:

$$\begin{aligned} \xi_{I1} &= \frac{R_P\left(I_{01} + I_{02}\right)}{n_1 V_{th}} e^{\frac{R_P\left(I_{PV} + I_{01} + I_{02} - I\right)}{n_1 V_{th}}}, \\ \xi_{I2} &= \frac{R_P\left(I_{01} + I_{02}\right)}{n_2 V_{th}} e^{\frac{R_P\left(I_{PV} + I_{01} + I_{02} - I\right)}{n_2 V_{th}}}. \end{aligned} \tag{45}$$

After that research, in 2016, Gao et al. [19] proposed the following analytical *I-V* expression:

$$I = \frac{R_P\left(I_{PV} + I_{01} + I_{02}\right) - V}{R_S + R_P} - r\frac{n_1 V_{th}}{2R_S} W\left(\xi_1\right) - \left(1 - r\right) \cdot \frac{n_2 V_{th}}{2R_S} W\left(\xi_2\right). \tag{46}$$

Individual terms ξ_{I1} and ξ_{I2} are defined as follows:

$$\begin{aligned} \xi_1 &= \frac{R_S R_P I_{01}}{r \cdot n_1 V_{th}\left(R_S + R_P\right)} e^{\frac{R_P\left(R_S I_{PV} + R_S I_{01}/r + V\right)}{n_1 V_{th}\left(R_S + R_P\right)}} \\ \xi_2 &= \frac{R_S R_P\left(I_{01} + I_{02}\right)}{\left(1 - r\right) \cdot n_2 V_{th}\left(R_S + R_P\right)} e^{\frac{R_P\left(R_S I_{PV} + R_S I_{02}/(1-r) + V\right)}{n_2 V_{th}\left(R_S + R_P\right)}}. \end{aligned} \tag{47}$$

Also, *r* is *the N*-vector and *N* is the number of measured *I-V* pairs computed for each pair of measured current I_m and voltage V_m values as follows:

$$r^{(i)} = \frac{I_{01}\left(\exp\left(\frac{{}^{(i)}V_m + {}^{(i)}I_m R_s}{n_1 V_{th}}\right) - 1\right)}{I_{01}\left(\exp\left(\frac{{}^{(i)}V_m + {}^{(i)}I_m R_s}{n_1 V_{th}}\right) - 1\right) + I_{02}\left(\exp\left(\frac{{}^{(i)}V_m + {}^{(i)}I_m R_s}{n_2 V_{th}}\right) - 1\right)}. \tag{48}$$

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In 2018, Chen et al. [20] proposed the following expressions:

$$I = \frac{R_P\left(I_{PV} + I_{01} + I_{02}\right) - V}{R_S + R_P} - \frac{V_{th}}{R_S}\left(\frac{k}{k+1}W\left(x_1\right) - \frac{1}{k+1}W\left(x_2\right)\right), \tag{49}$$

where x_1 and x_2 are defined as follows:

$$\begin{aligned} x_1 &= \frac{\frac{k}{k+1}R_S R_P I_{01}}{n_1 V_{th}\left(R_S + R_P\right)} e^{\frac{R_P\left(R_S\left(I_{PV} + \frac{\tau}{k}\right) + \frac{k}{k+1}R_S I_{01} + V\right)}{n_1 V_{th}\left(R_S + R_P\right)}}, \\ x_2 &= \frac{\left(k+1\right)R_S R_P I_{02}}{n_2 V_{th}\left(R_S + R_P\right)} e^{\frac{R_P\left(R_S\left(I_{PV} - \tau\right) + \left(k+1\right)R_S I_{02} + V\right)}{n_2 V_{th}\left(R_S + R_P\right)}}. \end{aligned} \tag{50}$$

Also, the dependence between currents of the diodes is defined with the following equation:

$$I_{D1} = k \cdot I_{D2} + \tau. \tag{51}$$

The unknown parameters k and τ can be calculated by using three different sets of solar cell *I-V* points – at short circuit, no load, and maximum power points.

In 2022, Ridha et al. [21] proposed the following expressions for solving DDM of the solar cell:

$$I = \frac{R_P\left(I_{PV} + I_{01} + I_{02}\right) - V}{R_S + R_P} - \frac{V_{th}}{R_S}\left(n_1 W\left(\xi_1\right) + n_2 W\left(\xi_2\right)\right), \tag{52}$$

where individual terms ξ_1 and ξ_2 are defined as follows:

$$\begin{aligned} \xi_1 &= \frac{R_S R_P I_{01}}{n_1 V_{th}\left(R_S + R_P\right)} e^{\frac{R_P\left(R_S I_{PV} + R_S I_{01} + V\right)}{n_1 V_{th}\left(R_S + R_P\right)}}, \\ \xi_2 &= \frac{R_S R_P I_{02}}{n_2 V_{th}\left(R_S + R_P\right)} e^{\frac{R_P\left(R_S I_{PV} + R_S I_{02} + V\right)}{n_2 V_{th}\left(R_S + R_P\right)}}. \end{aligned} \tag{53}$$

In the same work, the analytical *I-V* solution for TDM has been derived:

$$I = \frac{R_P\left(I_{PV} + I_{01} + I_{02}\right) - V}{R_S + R_P} - \frac{V_{th}}{R_S}\left(n_1 W\left(\xi_1\right) + n_2 W\left(\xi_2\right) + n_3 W\left(\xi_3\right)\right). \tag{54}$$

In the previous equation, terms ξ_1, ξ_2, and ξ_3 are defined as follows:

$$\begin{aligned} \xi_1 &= \frac{R_S R_P I_{01}}{n_1 V_{th}\left(R_S + R_P\right)} e^{\frac{R_P\left(R_S I_{PV} + R_S I_{01} + V\right)}{n_1 V_{th}\left(R_S + R_P\right)}}, \\ \xi_2 &= \frac{R_S R_P I_{02}}{n_2 V_{th}\left(R_S + R_P\right)} e^{\frac{R_P\left(R_S I_{PV} + R_S I_{02} + V\right)}{n_2 V_{th}\left(R_S + R_P\right)}}, \\ \xi_3 &= \frac{R_S R_P I_{03}}{n_3 V_{th}\left(R_S + R_P\right)} e^{\frac{R_P\left(R_S I_{PV} + R_S I_{03} + V\right)}{n_3 V_{th}\left(R_S + R_P\right)}}. \end{aligned} \tag{55}$$

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During 2024, Gao et al. [22] proposed the analytical *I-V* expression for multi-diode solar cells in terms of a single Lambert W function:

$$I=\frac{I_{PV}+\sum_{i=1}^{N_d} I_{0i}-\frac{V}{R_p}}{1+\frac{R_S}{R_p}}-n_1\frac{V_{th}}{R_S}W\left(\xi_1\right), \tag{56}$$

where ξ_1 is defined as follows:

$$\xi_1=\frac{R_S\sum_{i=1}^{N_d} p_i I_{0i}}{n_1V_{th}\left(1+\frac{R_S}{R_p}\right)}\cdot\exp\left(\frac{R_S\left(I_{pv}+\sum_{i=1}^{N_d} I_{0i}\right)+V}{n_1V_{th}\left(1+\frac{R_S}{R_p}\right)}\right). \tag{57}$$

Also, in terms of multiple Lambert functions, *I-V* expression can be derived as follows:

$$I=\frac{I_{PV}+\sum_{i=1}^{N_d} I_{0i}-\frac{V}{R_P}}{1+\frac{R_S}{R_P}}-\frac{V_{th}}{R_S}\sum_{i=1}^{N_d} r_i n_i W\left(\zeta_i\right), \tag{58}$$

where ζ_1 and r_i are defined as follows:

$$\begin{gathered}\zeta_1=\frac{R_s\frac{I_{0i}}{r_i}}{n_iV_{th}\left(1+1+\frac{R_S}{R_P}\right)}e^{\frac{R_s\left(I_{PV}+\frac{I_{0i}}{r_i}\right)+V}{n_iV_{th}\left(1+\frac{R_S}{R_P}\right)}},\\ r_i=\frac{I_{0i}\left(\exp\left(\frac{V+IRs}{n_iV_{th}}\right)-1\right)}{\sum_{j=1}^{N_d} I_{0j}\left(\exp\left(\frac{V+IRs}{n_jV_{th}}\right)-1\right)}.\end{gathered} \tag{59}$$

Additionally, Calasan et al. in 2024 [23] proposed two analytical approaches for triple diode solar cell *I-V* expressions. The first approach is described with the following equation:

$$\begin{aligned}I=&\frac{R_P\left(I_{PV}+I_{01}+I_{02}+I_{03}\right)-V}{R_P+R_s}-\frac{n_1V_{th}}{3R_s}W\left(\frac{R_sR_P\left(I_{01}+I_{02}+I_{03}\right)}{n_1V_{th}\left(R_P+R_s\right)}e^{\frac{R_P\left[R_S\left(I_{PV}+I_{01}+I_{02}+I_{03}\right)+V\right]}{n_1V_{th}\left(R_P+R_s\right)}}\right)\\&-\frac{n_2V_{th}}{3R_s}W\left(\frac{R_sR_P\left(I_{01}+I_{02}+I_{03}\right)}{n_2V_{th}\left(R_P+R_s\right)}e^{\frac{R_P\left[R_S\left(I_{PV}+I_{01}+I_{02}+I_{03}\right)+V\right]}{n_2V_{th}\left(R_P+R_s\right)}}\right)-\frac{n_3V_{th}}{3R_s}W\left(\frac{R_sR_P\left(I_{01}+I_{02}+I_{03}\right)}{n_3V_{th}\left(R_P+R_s\right)}e^{\frac{R_P\left[R_S\left(I_{PV}+I_{01}+I_{02}+I_{03}\right)+V\right]}{n_3V_{th}\left(R_P+R_s\right)}}\right).\end{aligned}$$

(60)

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On the other side, the second proposed approach in this work is presented as follows:

$$I = \frac{R_P\left(I_{PV}+I_{01}+I_{02}+I_{03}\right)-V}{R_P+R_s} - \frac{n_1V_{th}}{3R_s}W\left(\frac{R_sR_P\left(n_1V_{th}I_{01}+n_2V_{th}I_{02}+n_3V_{th}I_{03}\right)}{\left(n_1V_{th}\right)^2\left(R_P+R_s\right)}e^{\frac{R_P\left[R_S\left(I_{PV}+R_sI_{01}+R_sI_{02}+R_sI_{03}\right)+V\right]}{n_1V_{th}\left(R_P+R_s\right)}}\right)$$
$$-\frac{n_2V_{th}}{3R_s}W\left(\frac{R_sR_P\left(n_1V_{th}I_{01}+n_2V_{th}I_{02}+n_3V_{th}I_{03}\right)}{\left(n_2V_{th}\right)^2\left(R_P+R_s\right)}e^{\frac{R_P\left[R_S\left(I_{PV}+R_sI_{01}+R_sI_{02}+R_sI_{03}\right)+V\right]}{n_2V_{th}\left(R_P+R_s\right)}}\right)$$
$$-\frac{n_3V_{th}}{3R_s}W\left(\frac{R_sR_P\left(n_1V_{th}I_{01}+n_2V_{th}I_{02}+n_3V_{th}I_{03}\right)}{\left(n_3V_{th}\right)^2\left(R_P+R_s\right)}e^{\frac{R_P\left[R_S\left(I_{PV}+R_sI_{01}+R_sI_{02}+R_sI_{03}\right)+V\right]}{n_3V_{th}\left(R_P+R_s\right)}}\right).$$

(61)

1.1.5 Contribution to development of low carbon technologies, sustainability and circularity

The topic of this training course is the overview of various approaches for modeling solar cell, so it directly contributes to the development of low carbon technology – PV technology. Proper modeling of solar cells, in terms of the mathematical equations that describe the physics of the cell, directly contributes to increase in PV integration, and consequently reduction of greenhouse gases emission. In terms of electrical energy production via photovoltaic modules, the electrical output of a solar cell (modules and panels) is best represented by its current–voltage (*I*–*V*) characteristic curve. Accurate modeling of this curve is crucial and must consider all relevant factors, such as the inherent parameters of the cell and external influences, such as radiation (*G*) and temperature (*T*). The pronounced nonlinearity of the *I*–*V* characteristic means that various models are employed to represent the equivalent circuit of a solar cell, each differing in complexity and accuracy.

Therefore, the training course on modeling of the solar cells promotes the application of low carbon technologies by emphasizing adequate mathematical representations of most common used renewable source – photovoltaic/solar cell. The development and application of precise and reliable models ensures proper calculation of crucial characteristics of each PV cell, *I-V* and *P-V* characteristics.

1.1.6 Highlight on application in industry

The presentation of modeling approaches of solar cells presented in this course can find an application in industry, particularly in industry involved in renewable energy generation. Energy production from photovoltaic cells has increased, and therefore the issue of an adequate modeling of PV cells has become very important. Determining the relation between current and voltage on one side, and relation between produced power and voltage on the other side, is very important in order to determine basic characteristics of solar cells. Such obtained data serves as important input data

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for the industries dealing with installation of solar panels and electrical energy production from solar panels.

1.1.7 Contribution to development of skills and competences

This training course is designed to develop essential skills in the area of solar cells modeling approaches. Participants will gain a detailed analysis of physical processes in solar cells, and according to that, participants will be taught how to develop appropriate mathematical models. This course aims to present in details all mathematical equations used to define various solar cell models, and by that way ensure the participants will be able to implement the described mathematical models using any programming language.

After completing this course, participants will be equipped with a knowledge about existing models of solar cells and corresponding mathematical formulation of each model. Moreover, this course will teach the participants about improved solar cell models, that include various physical processes which occur in every solar cell. By that way, this course enhances critical thinking, as the participants might get an idea on how to additionally improve solar cell models and develop the mathematical formulation of any other phenomena that occurs during real operation mode of PV cells in industry applications.

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